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On the spin of gravitational bosons

D. V. Ahluwalia[a], N. Dadhich[b], M. Kirchbach[a],

[a] *Theoretical Physics Group, Fac. de Fisica de la UAZ, Zacatecas, Ap. Postal C-600, ZAC 98062, Mexico.*

[b] *Inter-University Center for Astronomy and Astrophysics (IUCAA), Post Bag 4, Ganeshkhind Pune 411 007, India.*

Abstract

We unearth spacetime structure of massive vector bosons, gravitinos, and gravitons. While the curvatures associated with these particles carry a definite spin, the underlying potentials cannot be, and should not be, interpreted as single spin objects. For instance, we predict that a spin measurement in the rest frame of a massive gravitino will yield the result 3/2 with probability one half, and 1/2 with probability one half. The simplest scenario leaves the Riemannian curvature unaltered; thus avoiding conflicts with classical tests of the theory of general relativity. However, the quantum structure acquires additional contributions to the propagators, and it gives rise to additional phases.

Email addresses: ahluwalia@heritage.reduaz.mx (D. V. Ahluwalia), nkd@iucaa.ernet.in (N. Dadhich), kirchbach@chiral.reduaz.mx (M. Kirchbach).

I. Interface of quantum and gravitational realms

The conceptual foundations of the general theory of relativity were established in an era when the quantum revolution had yet to fully inseminate the thinking of those desiring a unification of then-known interactions, or those working on a quantum theory of gravity. In that early part of the last century a crucial lesson from the unification of the electroweak theory was still several decades in the future. This circumstance has now given rise to a situation where a brute force quantization of gravity has exhausted the efforts of its pioneers, and may be considered to have failed. In the vacuum left by this failure two fundamentally new notions have arisen. These are the introduction of *extended objects* (strings, etc.) and *supersymmetry.* The former asks for abandoning the concept of point particles, while the latter is a natural extension of spacetime symmetries that places fermions and bosons at the same formal footing.

In recent years it has been realized that if one incorporates classical general-relativistic framework in certain *Gedanken* quantum measurement processes [1] then the wave-particle duality is modified [2,3,4]. For a one-dimensional motion, such a modification may be encoded in an expression of the form:

$$\lambda = \frac{\overline{\lambda}_P}{\tan^{-1}\left(\overline{\lambda}_P/\lambda_{dB}\right)} \begin{cases} \to \lambda_{dB} & \text{for low energy regime} \\ \to 4\lambda_P & \text{for Planck regime} \end{cases} , \qquad (1)$$

where

$$\lambda_P = \sqrt{\frac{\hbar G}{c^3}}, \quad \lambda_{dB} = \frac{h}{p}, \qquad (2)$$

are in turn the Planck length and the de Broglie wavelength; while $\overline{\lambda}_P = 2\pi\lambda_P$ is the Planck circumference.

The same set of *Gedanken* experiments implies a non-commutative nature of spacetime which has been extensively studied in a series of papers [5,6,7,8,9,10]. The modification of the wave-particle duality at the Planck scale renders the notion of a point particle operationally meaningless. Therefore, the latter must be replaced by some, yet undefined, *fuzzy spacetime entity,* which consistently captures in it the fundamental non-commutative nature of spacetime, and the modified wave-particle duality [of which Eq. (1) is only an approximate one-dimensional representation]. We shall argue in the concluding remarks that Eq. (1) can be studied in terrestrial laboratories. At the same time a recent Amelino-Camelia proposal allows to probe the associated spacetime fuzziness [11,12].

In order for the proposed fuzzy spacetime entities to possess well-defined

macroscopic properties they must transform from one inertial frame to another via various representations of the Lorentz group. The fuzzy nature *cannot* be captured by form factors so useful in describing QCD's extended objects such as protons, neutrons, and hadrons in general. The protons and the neutrons are described by Dirac's $(1/2, 0)\oplus(0, 1/2)$ representation space, and the remaining hadrons – irrespective of detailed QCD considerations – must also transform according one, or the other, representations of the Lorentz group. In addition, QCD solutions associated with these particles require relativistically covariant form factors that encode in them the extended nature of these objects. However, for the fuzzy spacetime entities the extendedness is characterized by the Planck length, λ_P, and it cannot be probed in the same manner as, e.g., a nucleon charge distribution. It is prevented by the general-relativistically modified wave-particle duality which saturates the matter wavelengths to λ_P (or, perhaps a few times λ_P).

At this stage a critical reader may ask: Are there *any* hints for the existence of fuzzy spacetime entities, and for supersymmetry? A purely formal answer to that question is: No. Nonetheless, elements of both spacetime fuzziness and supersymmetry seem to be present in particle physics:

(1) Spacetime fuzziness is hinted ever since the discovery of CP violation in the neutral kaon system. Also recent set of strong indications for flavor oscillations in the neutrino sector provide an independent support of this observation. Indeed, oscillation phenomena indicate towards the fact that – for reasons not yet fully understood – kaons and neutrinos are not mass eigenstates (to be identified with eigenstates of the first Casimir invariant of the Poincaré spacetime group). Their masses carry an inherent fundamental uncertainty, thus providing an example of a well-established fuzziness at the spacetime level that is consistent with principles of quantum framework.

(2) On the other side, in its simplest form, the algebra of supersymmetry contains besides the ten Poincaré group generators, also four anticommuting generators, the "supertranslations," which are components of a Majorana spinor. So, it comes about that spacetime structure of supersymmetry finds itself deeply intertwined with the Majorana aspect. In this context, recent results of Klapdor-Kleingrothaus *et al.* [13,14,15], which provide a first direct evidence for neutrinoless double beta decay, $0\nu\beta\beta$, are of particular interest. In its simplest interpretation, the $0\nu\beta\beta$ experimental signal arises from the Majorana nature of ν_e. Even though the experiment by itself does not necessarily require a supersymmetric framework for its explanation, the indication towards a Majorana spacetime structure suddenly acquires a pivotal importance as a structure at the heart of supersymmetry.

While fuzziness of spacetime has important and widely discussed consequences

for a quantum theory of gravity, and while this aspect is in fact a natural consequence of the interplay of the gravitational and quantum realms, it is our opinion that any successful framework for a quantum theory of gravity must incorporate a further lesson, namely the one taught by the unification of the electroweak theory. It is that very lesson on which the present essay is focused.

To briefly outline the task, recall that the massive gauge bosons of the electroweak theory – as regards their spacetime structure – have to transform according to the $(1/2,1/2)$ representation space,

$$\mathcal{A}^\mu(x): \quad (1/2,1/2)\,. \tag{3}$$

The latter, in being a four dimensional representation space, requires four independent degrees of freedom. Despite that demand, spacetime structure of massive gauge fields is treated within Proca's framework captured by the equation,

$$\partial_\mu \mathcal{F}^{\mu\nu}(x) + m^2 \mathcal{A}^\nu(x) = 0\,. \tag{4}$$

Due to the antisymmetric nature of the Proca curvature (or, more commonly called field strength tensor) $\mathcal{F}^{\mu\nu}(x)$, one in fact restricts to only three degrees of freedom. This happens because on taking the divergence of the above equation, and exploiting the stated antisymmetric nature of $\mathcal{F}^{\mu\nu}(x)$, the solutions are restricted to a subset of divergence-free $(1/2,1/2)$ space solutions, i.e. to $\mathcal{A}^\nu(x)$ that satisfy

$$\partial_\nu \mathcal{A}^\nu(x) = 0\,. \tag{5}$$

In this way, being mainly guided by the belief that the $(1/2,1/2)$ representations space, despite its obvious four-dimensional character, ought to carry only three physical degrees of freedom – Proca's framework attempts to force a single spin-one interpretation upon the $(1/2,1/2)$ representation space.

On the other side, the renormalizability of the electroweak theory required to supplement the Proca propagator by an additional – at that time ad hoc – Stückelberg term – which was thought to arise from an unusual scalar field of a negative norm outside the $(1/2,1/2)$ space under consideration. In addition, the propagator of this scalar had to be brought in with a "wrong" sign [16] relative to Proca's propagator in order to guarantee vanishing of divergences in the theory. This is well recounted by Veltman in his recent Nobel Prize lecture [16]. Unexpectedly, while its necessity is fully realized, the physical and mathematical origin of the Stückelberg term has not been understood at its fundamental representation-theory level.

So the purpose of this essay now is to trace back the origin of the Stückelberg term to the completeness of the $(1/2,1/2)$ representation space and to show how the electroweak gauge bosons come to carry an indefinite spin. Once we go through that argument it will be apparent to the reader how to move on to gravitinos and gravitons; and how they too cannot carry a definite spin as long as one allows them to be endowed with a non-zero mass (however small). [1] Note that the gauge bosons in a supersymmetric quantum theory of gravity are gravitinos and gravitons. From an observational point of view the data on pulsars PSR B1913+16 and PSR B1534+12 requires graviton mass to be less than 7.6×10^{-20} eV at 90% confidence [22], while the gravitino mass may be tens of orders of magnitude higher. The gravitinos and gravitons transform in turn as:

$$\psi^\mu(x): \quad \Big[(1/2,0) \oplus (0,1/2)\Big] \otimes (1/2,1/2)\,, \tag{6}$$
$$g^{\mu\nu}(x): \quad (1/2,1/2) \otimes (1/2,1/2)\,. \tag{7}$$

Before proceeding further, we wish to draw readers attention to a point that appears crucial to further understanding. We wish to emphasize that all the peculiarities of the vector potentials mentioned above carry relevance solely at the quantum level. Recall the classical work by Aharonov and Bohm [23] (for the massless limit), showing that the potentials give rise to non-trivial and observable quantum phases which do not have any classical counterpart. Moreover, the indefinite spin of the vector potentials so important for bringing into the propagator the terms needed for renormalizability, contrasts the case of $\mathcal{F}^{\mu\nu}(x)$, that is endowed by a unique spin, i.e., with spin one. As long as $\mathcal{F}^{\mu\nu}(x)$ encodes the forces acting on test particles, the multi-spin character of the vector potentials will leave the classical level of the theory unaltered.

II. Mathematical structure of the $(1/2,1/2)$ representation space

Spinors and Lorentz vectors play a pivotal role in physics. The latter embody in them the transformation properties of the $(1/2,1/2)$ representation space which, by definition, is the direct product of the $(1/2,0)$ and $(0,1/2)$ representation spaces:

$$(1/2,1/2): \quad (1/2,0) \otimes (0,1/2)\,. \tag{8}$$

We shall work in the momentum space.

[1] Already, graviton masses and lower-spin components in gravity are of interest in astrophysical and cosmological contexts; for instance, due to indications for an accelerating universe [17,18,19,20,21].

In the rest frame, where the three-momentum of the particle under consideration is zero, $\mathbf{p} = \mathbf{0}$; the $(1/2, 1/2)$ representation space decomposes into two subspaces of spin one, and spin zero. These subspaces are spanned by four vectors:

$$\mathcal{A}_a^{1,+1}(\mathbf{0}) = h^+ \otimes h^+ \,, \tag{9}$$
$$\mathcal{A}_a^{1,0}(\mathbf{0}) = \frac{1}{\sqrt{2}} \left(h^+ \otimes h^- + h^- \otimes h^+ \right) \,, \tag{10}$$
$$\mathcal{A}_a^{1,-1}(\mathbf{0}) = h^- \otimes h^- \,, \tag{11}$$
$$\mathcal{A}_a^{0,0}(\mathbf{0}) = \frac{1}{\sqrt{2}} \left(h^+ \otimes h^- - h^- \otimes h^+ \right) \,. \tag{12}$$

In Eqs. (9)-(12), $h^\pm$ are eigenstates of the spin-1/2 helicity operator, $(\boldsymbol{\sigma}/2) \cdot \widehat{\mathbf{p}}$

$$h^+ = m^{1/4} \begin{pmatrix} \cos(\theta/2) e^{-i\phi/2} \\ \sin(\theta/2) e^{i\phi/2} \end{pmatrix} \,, \quad h^- = m^{1/4} \begin{pmatrix} \sin(\theta/2) e^{-i\phi/2} \\ -\cos(\theta/2) e^{i\phi/2} \end{pmatrix} \,. \tag{13}$$

Here, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$; σ_i are the usual Pauli matrices, and $\widehat{\mathbf{p}}$ is the unit momentum vector with Cartesian components $(\sin(\theta)\cos(\phi),\ \sin(\theta)\sin(\phi),\ \cos(\theta))$. The factor $m^{1/4}$, besides other matters, allows, in the massless limit, the $\mathcal{A}_a^{s,h}(\mathbf{0})$ to identically vanish. Since massless particles have no rest frame, the preceding is a physical requirement. The subscript, a, is a Lorentz index (however, see Eq. (21) below). The superscripts are defined as:

$$\mathbf{S}^2\, \mathcal{A}_a^{s,h}(\mathbf{0}) = s(s+1)\, \mathcal{A}_a^{s,h}(\mathbf{0}) \,, \quad \mathbf{S} \cdot \widehat{\mathbf{p}}\, \mathcal{A}_a^{s,h}(\mathbf{0}) = h\, \mathcal{A}_a^{s,h}(\mathbf{0}) \,. \tag{14}$$

The generators of rotation for the $(1/2, 1/2)$ representation space [that appear in Eq. (14)] are:

$$S_x = \frac{1}{2} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \,, \; S_y = \frac{1}{2} \begin{pmatrix} 0 & -i & -i & 0 \\ i & 0 & 0 & -i \\ i & 0 & 0 & -i \\ 0 & i & i & 0 \end{pmatrix} \,, \; S_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \,. \tag{15}$$

The application of the boost [2]

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[2] The $(1/2, 0)$- and $(0, 1/2)$- boosts that appear in the equation below are:

$$\kappa^{(\frac{1}{2},0)} = \exp\left(+ \frac{\boldsymbol{\sigma}}{2} \cdot \boldsymbol{\varphi} \right) = \sqrt{\frac{E+m}{2\,m}} \left(1_2 + \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m} \right) \,,$$

$$\kappa^{\left(\frac{1}{2},\frac{1}{2}\right)} = \kappa^{\left(\frac{1}{2},0\right)} \otimes \kappa^{\left(0,\frac{1}{2}\right)}, \tag{16}$$

to the $\mathcal{A}_a^{s,h}(\mathbf{0})$ yields, in the order presented in Eqs. (9)-(12),

$$A_a(\mathbf{p},\,1) = \frac{\sqrt{m}}{2}\begin{pmatrix} 2e^{-i\phi}\cos^2(\theta/2) \\ \sin(\theta) \\ \sin(\theta) \\ 2e^{i\phi}\sin^2(\theta/2) \end{pmatrix}, \quad \mathcal{A}_a(\mathbf{p}\,,2) = \frac{1}{\sqrt{2\,m}}\begin{pmatrix} e^{-i\phi}E\sin(\theta) \\ -\left(|\mathbf{p}| + E\cos(\theta)\right) \\ |\mathbf{p}| - E\cos(\theta) \\ -e^{i\phi}E\sin(\theta) \end{pmatrix},$$

$$\mathcal{A}_a(\mathbf{p}\,,3) = \frac{\sqrt{m}}{2}\begin{pmatrix} 2e^{-i\phi}\sin^2(\theta/2) \\ -\sin(\theta) \\ -\sin(\theta) \\ 2e^{i\phi}\cos^2(\theta/2) \end{pmatrix}, \quad \mathcal{A}_a(\mathbf{p},\,4) = \frac{1}{\sqrt{2\,m}}\begin{pmatrix} e^{-i\phi}|\mathbf{p}|\sin(\theta) \\ -\left(E + |\mathbf{p}|\cos(\theta)\right) \\ E - |\mathbf{p}|\cos(\theta) \\ -e^{i\phi}|\mathbf{p}|\sin(\theta) \end{pmatrix}. \tag{17}$$

The reason for change in notation from $\mathcal{A}^{s,h}(\mathbf{0})$ to $\mathcal{A}(\mathbf{p},\,\zeta)$, $\zeta = 1, 2, 3, 4$, shall be made apparent in Sec. III.1 below.

In a parallel to the Dirac's $(1/2, 0) \oplus (0, 1/2)$ representation space, and to emphasize certain non-trivial mathematical similarities, we introduce

$$\lambda^{00} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{18}$$

and define:

$$\overline{\mathcal{A}}_a(\mathbf{p}) = \left(\mathcal{A}_a\right)^\dagger \lambda^{00}\,. \tag{19}$$

$$\kappa^{\left(0,\frac{1}{2}\right)} = \exp\left(-\frac{\boldsymbol{\sigma}}{2}\cdot\boldsymbol{\varphi}\right) = \sqrt{\frac{E+m}{2\,m}}\left(1_2 - \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+m}\right).$$

The boost parameter, $\boldsymbol{\varphi}$, is defined as:

$$\cosh(\varphi) = \frac{E}{m}, \quad \sinh(\varphi) = \frac{|\mathbf{p}|}{m}, \quad \widehat{\boldsymbol{\varphi}} = \frac{\mathbf{p}}{|\mathbf{p}|}\,.$$

We use the notation in which 1_n and 0_n represent $n \times n$ identity and null matrices, respectively.

Then, the orthonormality and completeness relations for the $(1/2,1/2)$ representation space read (with no summation intended on the Lorentz index):

$$\overline{\mathcal{A}}_a(\mathbf{p}\,,\zeta)\,\mathcal{A}_a(\mathbf{p}\,,\zeta') = \begin{cases} -\,m\,\delta_{\zeta\zeta'}, & \text{for } \zeta = 1,2,3 \\ +\,m\,\delta_{\zeta\zeta'}, & \text{for } \zeta = 4 \end{cases}\,,$$
$$\frac{1}{m}\left[\mathcal{A}_a(\mathbf{p},\,4)\,\overline{\mathcal{A}}_a(\mathbf{p}\,,4) - \sum_{\zeta=1,2,3}\mathcal{A}_a(\mathbf{p},\,\zeta)\,\overline{\mathcal{A}}_a(\mathbf{p}\,,\zeta')\right] = 1_4\,. \tag{20}$$

The Lorentz index, a, that appears in the above expressions is not the usual "time, space" (i.e., usual 0,1,2,3) index. The latter, denoted by $\mu,\nu\ldots$, is obtained via the following transformation,

$$\mathcal{A}^\mu(\mathbf{p}) = \mathcal{S}^{\mu a}\mathcal{A}_a(\mathbf{p})\,, \tag{21}$$

with

$$\mathcal{S} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & i & -i & 0 \\ -i & 0 & 0 & i \\ 1 & 0 & 0 & 1 \\ 0 & i & i & 0 \end{pmatrix}\,. \tag{22}$$

These satisfy a new wave equation [24]:

$$\left(\Lambda_{\mu\nu}p^\mu p^\nu \pm m^2 I_4\right)\mathcal{A}(\mathbf{p}\,,\zeta) = 0\,, \tag{23}$$

where the plus sign is to be taken for $\zeta = 1,2,3$, while the minus sign is for $\zeta = 4$.

The $\Lambda_{\mu\nu}$ matrices are: $\Lambda_{00} = \mathrm{diag}(1,-1,-1,-1)$, $\Lambda_{11} = \mathrm{diag}(1,-1,1,1)$, $\Lambda_{22} = \mathrm{diag}(1,1,-1,1)$, $\Lambda_{33} = \mathrm{diag}(1,1,1,-1)$, and

$$\Lambda_{01} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\,,\quad \Lambda_{02} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\,,\quad \Lambda_{03} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}\,,$$

$$\Lambda_{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; \Lambda_{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \; \Lambda_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{24}$$

They are symmetric in the Lorentz index, $\Lambda_{\mu\nu} = \Lambda_{\nu\mu}$.[3] The interchange, $\mathcal{A}^\mu(\mathbf{p},\, \zeta = 1,2,3) \rightleftharpoons \mathcal{A}^\mu(\mathbf{p},\, \zeta = 4)$, corresponds to

$$m \rightleftharpoons i\,m\,, \tag{25}$$

This circumstance suggests that the "negative mass squared" term in spontaneous symmetry breaking may have the above spacetime structure at its origin.

III. Physical structure of the $(1/2, 1/2)$ representation space

1. The $\mathcal{A}(\mathbf{p},\, \zeta)$, are, in general, not eigenstates of $\mathbf{S}^2$. This is because the $\kappa^{\left(\frac{1}{2},\frac{1}{2}\right)}$ does *not,* for an arbitrary $\mathcal{A}(\mathbf{p}\,)$, commute with $\mathbf{S}^2$.

If spin is to be identified with eigenvalues of $\mathbf{S}^2$, then $(1/2, 1/2)$ representation space does not carry a single-spin interpretation. Spin-1 and Spin-0 thus become covariantly inseparable in the $(1/2, 1/2)$ representation space. On the other hand, if one wishes to have a *pure* spin-1 massive object, then that object must transform according to the $(1, 0) \oplus (0, 1)$ representation [25].

Even though, in their rest frame $\mathcal{A}(\mathbf{p},\, \zeta = 1, 2, 3)$ and $\mathcal{A}(\mathbf{p}, \zeta = 4)$ appear as spin one and spin zero objects, respectively; they cannot be identified with spin one and "scalar (i.e. spin 0)" particles in an arbitrary frame.

2. The $\mathcal{A}(\mathbf{p},\, \zeta = 1, 2, 3)$, coincide with the solutions of Proca framework (and are divergence-free); whereas $\mathcal{A}(\mathbf{p},\, \zeta = 4)$, is divergence-full:

$$p_\mu \mathcal{A}^\mu(\mathbf{p},\, \zeta = 1, 2, 3) = 0\,, \tag{26}$$
$$p_\mu \mathcal{A}^\mu(\mathbf{p},\, \zeta = 4) = i m^{3/2}\,. \tag{27}$$

Thus mass serves as a source of $\mathcal{A}^\mu(\mathbf{p},\, \zeta = 4)$.

3. On quantization, we find that the *numerator* of the propagator associated with the $\zeta = 1, 2, 3$ sector (Proca sector) is:

[3] Parenthetically, we note that the $\mathcal{S}$-transformed λ^{00}, i.e., $\mathcal{S}\lambda^{00}\mathcal{S}^{-1}$, equals Λ_{00} and is nothing but the standard flat-spacetime metric, $\mathbf{g}$, with the signature $(1, -1, -1, -1)$.

$$\frac{1}{m}\sum_{\zeta=1,2,3}\mathcal{S}\left(\mathcal{A}^\mu(\mathbf{p},\zeta)\,\overline{\mathcal{A}}^\nu(\mathbf{p},\,\zeta)\lambda^{00}\right)\mathcal{S}^{-1} = -\,g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2} \tag{28}$$

while the contribution to the numerator of the propagator from the $\zeta = 4$ sector (Stückelberg sector) turns out to be:

$$\frac{1}{m}\mathcal{S}\left(\mathcal{A}^\mu(\mathbf{p},\,4)\,\overline{\mathcal{A}}^\nu(\mathbf{p},\,4)\lambda^{00}\right)\mathcal{S}^{-1} = \quad \frac{p^\mu p^\nu}{m^2} \tag{29}$$

The latter contribution lies outside the Proca framework, and is the *key* ingredient in the renormalizability of the electroweak unification. Here it appears naturally – with the same physical interpretation that mass is its source. Here, as well as in the electroweak unification, the relative sign of the two contributions is opposite to that which naive vacuum expectation value of the relevant time-ordered field operators implies.

4. The $\mathcal{A}^\mu(\mathbf{p},\,\zeta = 1,3)$, correspond to left- and right- circular polarizations, while the $\mathcal{A}^\mu(\mathbf{p},\,\zeta = 2,4)$ are the longitudinal, and time-like polarizations, respectively.

In order to construct the field strength tensor, $\mathcal{F}^{\mu\nu}(\mathbf{p})$, we provide the relation between, $\mathcal{A}^\mu(\mathbf{p},\,\zeta)$, and the standard polarization vectors, $e^\mu(\mathbf{p},\ell)$,

$$\begin{aligned}
&e^\mu(\mathbf{p},\,\perp_1) = \frac{i}{\sqrt{2}}\Big(A^\mu(\mathbf{p},\,3) - A^\mu(\mathbf{p},\,1)\Big)\,,\\
&e^\mu(\mathbf{p},\,\perp_2) = -\,\frac{1}{\sqrt{2}}\Big(A^\mu(\mathbf{p},\,3) + A^\mu(\mathbf{p},\,1)\Big)\,,\\
&e^\mu(\mathbf{p},\,\|) = iA^\mu(\mathbf{p},\,2)\,,\\
&e^\mu(\mathbf{p},\,0) = iA^\mu(\mathbf{p},\,4)\,,
\end{aligned} \tag{30}$$

where the symbols $\perp_1$ and $\perp_2$ represent two mutually orthogonal polarizations perpendicular to $\mathbf{p}$, while the $\|$ labels polarizations along the $\mathbf{p}$, and 0 represents a time-like polarization. The matrix of the momentum-space field strength is found to be,[4]

$$\mathcal{F}(\mathbf{p},\,\ell) = e(\mathbf{p},\,\ell)\;e(\mathbf{p},\,0)^\dagger - e(\mathbf{p},\,0)\;e(\mathbf{p},\,\ell)^\dagger\,. \tag{31}$$

where $\ell = \perp_1, \perp_2, \|, 0$. This definition satisfies

$$\mathcal{F}(\mathbf{p},\,\ell)\,\mathbf{g}\,e(\mathbf{p},\,0) = m\;e(\mathbf{p},\,\ell)\,, \quad \ell = \perp_1, \perp_2, \|\,. \tag{32}$$

[4] As is apparent from Table 1, $e^\mu(\mathbf{p},\,0)$ is nothing but $(1/\sqrt{m})\;p^\mu$. Also, of interest is to note that the factors of, i, in Eqs. (30), make $e^\mu(\mathbf{p},\,\ell)$ real.

ℓ	$e(\mathbf{p}\,,\ell)$	$\mathcal{F}(\mathbf{p}\,,\ell)$
$\perp_1$	$\sqrt{m}\begin{pmatrix} 0 \\ -c_\phi c_\theta \\ -s_\phi c_\theta \\ s_\theta \end{pmatrix}$	$\begin{pmatrix} 0 & E\,c_\phi c_\theta & E\,c_\theta s_\phi & -E\,s_\theta \\ -E\,c_\phi c_\theta & 0 & 0 & -p\,c_\phi \\ -E\,c_\theta s_\phi & 0 & 0 & -p\,s_\phi \\ E\,s_\theta & p\,c_\phi & p\,s_\phi & 0 \end{pmatrix}$
$\perp_2$	$\sqrt{m}\begin{pmatrix} 0 \\ s_\phi \\ -c_\phi \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -E\,s_\phi & E\,c_\phi & 0 \\ E\,s_\phi & 0 & p\,s_\theta & p\,c_\theta\,s_\phi \\ -Ec_\phi & -p\,s_\theta & 0 & -p\,c_\phi c_\theta \\ 0 & -p\,c_\theta\ s_\phi & p\,c_\phi\,c_\theta & 0 \end{pmatrix}$
$\parallel$	$\frac{1}{\sqrt{m}}\begin{pmatrix} p \\ E\,c_\phi\ s_\theta \\ E\,s_\phi\ s_\theta \\ E\,c_\theta \end{pmatrix}$	$\begin{pmatrix} 0 & -m\,c_\phi\,s_\theta & -ms_\phi\,s_\theta & -mc_\theta \\ m\,c_\phi\,s_\theta & 0 & 0 & 0 \\ m\,s_\phi\,s_\theta & 0 & 0 & 0 \\ m\,c_\theta & 0 & 0 & 0 \end{pmatrix}$
0	$\frac{1}{\sqrt{m}}\begin{pmatrix} E \\ p\,c_\phi s_\theta \\ p\,s_\phi s_\theta \\ p\,c_\theta \end{pmatrix}$	$\begin{pmatrix} 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \end{pmatrix}$

Table 1
Field strength tensor in momentum space for each of the polarization vectors. We have used the abbreviations: $\cos(x) = c_x$, $\sin(x) = s_x$, $p = |\mathbf{p}|$. The $e^\mu(\mathbf{p},\,\ell)$ are divergence-free for $\ell = \perp_1, \perp_2, \|$, while the contrary is true for $\ell = 0$: $p_\mu e^\mu(\mathbf{p},\,\ell) = 0$, for $\ell = \perp_1, \perp_2, \|$; while, $p_\mu e^\mu(\mathbf{p},\,\ell) = m^{3/2}$, for $\ell = 0$.

The above equation is equivalent to the Proca equation in momentum space.

The explicit expressions for $\epsilon^\mu(\mathbf{p},\,\ell)$, and the corresponding $\mathcal{F}^{\mu\nu}(\mathbf{p})$, are summarized in Table 1.

There is nothing in our formalism which requires to identify $\mathcal{F}(\mathbf{p}\,,\ell)$ with massive electrodynamics. However, doing so allows to make a few consistency tests and to gain a few useful insights. Thus, identifying the $\mathcal{F}(\mathbf{p}\,,\ell)$ matrix as following general matrix,

$$
\mathcal{F}^{\mu\nu}(\mathbf{p}) = \begin{pmatrix} 0 & E_x(\mathbf{p}) & E_y(\mathbf{p}) & E_z(\mathbf{p}) \\ -E_x(\mathbf{p}) & 0 & -B_z(\mathbf{p}) & B_y(\mathbf{p}) \\ -E_y(\mathbf{p}) & B_z(\mathbf{p}) & 0 & -B_x(\mathbf{p}) \\ -E_z(\mathbf{p}) & -B_y(\mathbf{p}) & B_x(\mathbf{p}) & 0 \end{pmatrix}, \tag{33}
$$

it is apparent that the following consistency tests are satisfied:

(1) From Table 1, we immediately infer that all components of the $\mathbf{B}(\mathbf{p})$ fields are proportional to the magnitude of $\mathbf{p}$. Thus, it verifies that without currents, i.e. with $\mathbf{p}=\mathbf{0}$, there are no $\mathbf{B}$ components in $\mathcal{F}(\mathbf{p}\,,\ell)$.
(2) In the massless limit, the longitudinal $\mathcal{F}(\mathbf{p}\,,\ell)$, i.e. $\mathcal{F}(\mathbf{p}\,,\ell=\|)$, identically vanishes.
(3) In the massless limit, setting $\theta=\pi/2$, and $\phi=\pi/2$, yields the expected $\mathcal{F}(\mathbf{p}\,,\ell)$ for an electromagnetic wave propagating along the y-axis. The resulting $\mathcal{F}(\mathbf{p}\,,\perp_1)$ (and $\mathcal{F}(\mathbf{p}\,,\perp_2)$) contain $\mathbf{E}$ and $\mathbf{B}$ fields which are respectively along the z (x)- and x (z)- axes. Furthermore, they carry equal magnitudes.

4. A conjecture and concluding remarks

Having gathered together all the essential elements we come to the final task of this essay. We assert that the essential result, and the lesson to be learned from the renormalizable electroweak theory, is that massive gauge theories based upon Proca's framework are incomplete. On the one side, the renormalizability of the electroweak theory demands that Proca framework be supplemented by the Stückelberg sector. The latter, in the usual framework, is introduced via a scalar field (or, fields). On the other side, completeness in $(1/2,1/2)$ requires it to also consist of two sectors. At rest, one of them, the time-like polarization vector, behaves as a scalar, while the remaining three degrees of freedom constitute an ordinary spin-one vector. In boosted inertial frames, the former "scalar" from the rest frame transforms into a state that is no longer of specified spin and that does not transform according to the trivial representation of the Lorentz group. It is that very state that produces the Stückelberg term. It is worth emphasizing that the time-like polarizations associated with $\mathcal{A}^\mu(\mathbf{p},4)$, or, equivalently with $e^\mu(\mathbf{p},0)$, do not contribute to $\mathcal{F}^{\mu\nu}(\mathbf{p}\,,\ell)$. That is, the Stückelberg sector does not produce classical forces. Its importance lies in its contribution to quantum phases, to the propagator, and its role in renormalizability of massive gauge theories.

The relevant representation space for gravitational phenomena – dictated by the spacetime metric – in being a direct product of two $(1/2,1/2)$ representation spaces, in local inertial frames, carries counterparts of all the above-indicated mathematical and physical elements.

Quantum theory of gravity should be expected to be a supersymmetric theory of fuzzy spacetime entities with a set of gravitational gauge bosons. The latter set contains, at the very least, massive gravitinos, and nearly massless gravitons as dictated by observational data.

A detailed spacetime structure of massive gravitinos can be found in Ref. [24]. There it is argued that spin measurement on a massive gravitino shall yield not only the expected spin component of spin $3/2$, but also an additional set of components carrying spin $1/2$. These components, on the basis of the lessons learned from the electroweak theory, should be expected to carry significance for the renormalizability of the theory.

The other gravitational gauge boson, graviton, transforms as a massive $(1/2, 1/2)\otimes(1/2, 1/2)$ particle without projecting out the lower spin components – i.e., the Stückelberg counterparts. In keeping the lower spin components of the massive gravitational bosons, we produce a framework which differs from the one to which the van Dam-Veltman considerations apply [26,27]. Such a field, in its rest frame, contains 16 degrees of freedom (dof) distributed over a single spin-2 component (5 dof), three spin-1 sectors (9 dof), and two spin-0 components (2 dof). Furthermore for CPT invariance one must also incorporate the charge conjugated degrees of freedom. Thus, in a CPT covariant structure of the massive $(1/2, 1/2) \otimes (1/2, 1/2)$ contains a spin-2 component with 10 dof, three spin-1 sectors with 18 dof, and two spin-0 components with 4 dof.

Once the van Dam-Veltman observations no longer apply, one may look at the graviton as an object described by a massive $(1/2, 1/2) \otimes (1/2, 1/2)$ representation space. The graviton, then, is not a single spin object, but it carries in it the several spin components. This is in exact parallel of the gauge bosons of the electroweak theory where the bosons cannot be seen as pure spin objects (except in their rest frame).

We thus make the following conjecture:

> The renormalizability of the quantum theory of gravity would require that the gravitational bosons, gravitinos and graviton, be treated as multi-spin particles in the sense defined above.

This conjecture has some remarkable additional consequences. To see this, first note that despite the fact that the full $(1/2, 1/2)$ representation space is spanned by particles which do not carry a definite spin as encoded in the spin content of the $\mathcal{A}^\mu(x)$, the associated curvature, as encoded in the $\mathcal{F}^{\mu\nu}(x)$, the field strength tensor, is a pure spin one object. Similarly, despite the fact that the massive graviton, transforming as $(1/2, 1/2) \otimes (1/2, 1/2)$ representation space, carries several spin components, the associated field strength tensor $\mathcal{R}^{\mu\nu\lambda\sigma}(x)$ – i.e. the Riemannian curvature tensor – is a pure spin-2 object.

The incomplete treatments of the $\mathcal{A}^\mu(x)$, as well as $\psi^\mu(x)$ and $g^{\mu\nu}(x)$, either ignore, or project out, lower spin components. However, the latter are natural inhabitants of these spaces. Yet, this circumstance does not affect the induced curvatures, $\mathcal{F}(x)$ and $\mathcal{R}(x)$. As such, as far as measured forces are concerned the noted incompleteness carries no significance. The heuristic treatments of $(1/2, 1/2)$, as well as $(1/2, 1/2)\otimes(1/2, 1/2)$ yield correct electroweak and gravitational *forces*. The lower spin components enter entirely at the quantum level – without inducing forces – by their contribution residing in certain phases, and via their contributions to the propagator that makes the theories renormalizable. The latter phases can be studied via neutrino oscillations, or a set of orthogonal states in linear superposition of different energy/mass eigenstates [28,29,30,31,32,33]. [5]

Thus a quantum theory of gravity lives in a non-commutative spacetime, and in fact is its theory. Furthermore, even when the non-commutative structure of spacetime can be neglected, the conceptual framework requires attention not only to forces that are acted upon test states, but one must also pay due attention to existence of certain non-trivial gravitationally-induced quantum phases. Under certain circumstances the former may be zero, while the latter may be non-vanishing and observable. To run the point home, first recall that local density fluctuations in the cosmological context can, and do, create regions characterizable by a set of dimensionless gravitational potentials. The latter have a characteristic dimensionless value of about $|\Phi_0| \sim 10^{-5}$, and are, in general, several order of magnitude larger than those arising from stars and planets that may inhabit these regions. In our solar system the (magnitude of) dimensionless lunar gravitational potential is 3.14×10^{-11}, for Earth it is 6.95×10^{-10}, while for Sun it is 2.12×10^{-6}. Next, note that while the planetary and lunar orbits are determined by gradients in these latter potentials; the Φ_0, to a good approximation, essentially has the affect of red-shifting the orbital periods. However, for quantum systems embedded in quantum/classical gravitational fields one may entertain a violation of the equivalence principle (VEP) with observable phenomenological effects [38,39,40]. In such scenarios, the essentially force-free $|\Phi_0| \sim 10^{-5}$ amplifies the terrestrial observability of VEP by about five orders of magnitude.

In reference to Eq. (1), one need not await to reach Planck energies in terrestrial accelerators. Quantum states carrying Planck mass can be easily created and studied in laboratory using superconducting quantum interference devices

[5] For an early work on gravitationally induced phases in neutron interferometry [34], see Ref. [35]. With notable exceptions of Ref. [36,37], most early works on gravitationally induced phases were devoted to single mass eigenstates. The above-quoted references address themselves to states in linear superposition of different mass eigenstates. It allows to probe certain additional, and non-trivial, gravitationally induced phases.

(SQUID). In these devices the mass carried by the superconducting quantum state is,

$$M_{SQUID} \sim f(T)\, m_c\, N_A\,, \tag{34}$$

where m_c is the mass of a Cooper pair, N_A is the Avogadro number, and $f(T)$ is the fraction of the electrons that are in a superconducting state. Since all the Copper pairs are part of a single superconducting state, and $f(T)$ can reach close to unity at temperatures, T, sufficiently below the critical temperature, M_{SQUID} becomes of the order of Planck mass. This fact has apparently escaped attention. But it may carry significance for theorists as well as experimentalists to probe the interface of the gravitational and quantum realms in the emerging field of experimentally-driven quantum gravity phenomenology.

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